Estimating Biomass and Management Parameters from Length Composition Data: A Stock Assessment Method for Data Deficient Situations

Bernard A. Megrey and Chang Ik Zhang

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Outline

• Motivation
• Model description
• Compare a numbers-based length cohort analysis (Jones LCA, 1979) method to a new biomass-based method that explicitly incorporates growth.
• Investigate the performance of the biomass-based LCA and the more traditional numbers-based LCA on simulated data.
• Demonstrate management applications.
• Test performance by applying the model to actual data on eastern Bering Sea northern rock sole
Motivation

• Long time series of catch are not always available.
• Small fish populations are not usually assessed with research surveys.
• Often catch is recorded in weight and by size groups, but no age data are collected.
• FAO (2005) reports that 143 exploited stocks (20%) are not assessed due to lack of available information.
• These situations are exactly those that describe small-scale or artisanal fisheries.
• Stocks still need active management to maintain sustainability.
Objective

- Describe a biomass-based cohort analysis method based on length composition data (LCA) that can be used in small-scale fisheries situations.
- Develop model extensions to allow the calculation of relevant management metrics using only length composition data.
- Apply to data of an exploited and managed stock (i.e. assessments and research surveys performed)
### Typical LCA Calculations

<table>
<thead>
<tr>
<th>Step</th>
<th>Number-based LCA</th>
<th>Biomass-based LCA</th>
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<tbody>
<tr>
<td>1</td>
<td>$CN = \frac{CW}{\bar{W}}$</td>
<td>$C_i^W = CW \cdot p_i^W$</td>
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<td>2</td>
<td>$C_i^N = CN \cdot p_i^N$</td>
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<tr>
<td>3</td>
<td>$\hat{N}<em>i = fn(C_i^N, M, K, L</em>\infty)$</td>
<td>$\hat{B}<em>i = fn(C_i^W, M, K, L</em>\infty, W_i)$</td>
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<tr>
<td>4</td>
<td>$\hat{B}_i = N_i \cdot W_i$</td>
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</table>

- $CN$ – catch in number
- $CW$ – catch in weight
- $\bar{W}$ – average weight
- $N_i$ – number at length
- $B_i$ – biomass at length
- $p_i^N$ – proportion of catch in number-at-length
- $p_i^W$ – proportion of catch in weight-at-length
- $C_i^N$ – catch in number-at-length
- $C_i^W$ – catch in weight-at-length
- $W_i$ – weight at length
Problems with Numbers-based LCA

• In the Jones numbers-based method, catch weight is converted to numbers, abundance is estimated in numbers, and then population numbers are converted back to weight (biomass) for management actions (i.e. TAC, quota etc).

• The first and last step introduce errors into the population estimates.

• The first and last step can be eliminated by directly using catch that is given in weight-at-length to estimate biomass-at-length.

• Numbers-based methods assume mortality (Z) is the only process affecting biomass. Even if Z=0, growth (G) affects changes in biomass.

• Numbers-based methods will ALWAYS overestimate biomass when growth is positive.
5 Data Requirements

Data Requirements from Fishery

• 1. Length-frequency data. Weight at length. Catch length composition (catch biomass by length interval) for one harvest year minimum.

• 2. Total catch biomass (one harvest year minimum).
Data Requirements-General

- 3. Length-Weight Data
- **parameters**: Allometric length-weight parameters \((\alpha, \beta)\)
- **data**: length, weight

\[ W_l = \alpha L_l^\beta \]
Data Requirements-General

• 4. Length-at-Age Data
• parameters: von Bertalanffy parameters ($K, L_\infty, t_0$)
• data: length, age

$$L_t = L_\infty (1 - e^{(-K(t-t_0))})$$
Data Requirements

• 5. Natural Mortality (M)

• Use empirical relationship based on life history parameters


• $t_{mb} = fxn(t_{max})$

\[
M = \frac{\beta K}{e^{K(t_{mb} - t_0)} - 1}
\]
The Model

1. The generalized equation for change in biomass (indexing backwards in time) is

\[
B_t = B_{t+\Delta t} \exp(M \cdot \Delta t - G_t) + C_t \exp\left(\frac{M \cdot \Delta t - G_t}{2}\right)
\]

2. We can solve the von Bertalanffy growth equation for \( t \)

\[
L_t = L_\infty \left(1 - e^{-K(t-t_0)}\right)
\]

\[
t_{l_i} = t_0 - \frac{1}{K} \ln\left(\frac{L_\infty - l_i}{L_\infty}\right)
\]

3. If \( \Delta t \) is the time to grow from length class \( l_i \) to length class \( l_i + \Delta l \) then

\[
t_{l_i+\Delta l} = t_0 - \frac{1}{K} \ln\left(\frac{L_\infty - l_{i+\Delta l}}{L_\infty}\right)
\]

\[
\Delta t_{l_i} = t_{l_i+\Delta l} - t_{l_i} = \frac{1}{K} \ln\left(\frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}}\right)
\]
The Model (con’t)

4. Substituting 3 into 1 gives

\[ B_{l_i} = B_{l_i + \Delta l} \exp\left( \frac{M}{K} \ln\left( \frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}} \right) - G_i \right) + C_i \exp\left( \frac{M}{2K} \ln\left( \frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}} \right) - \frac{G_i}{2} \right) \]

5. Which simplifies to

\[ B_{l_i} = \left( B_{l_i + \Delta l} X_{l_i} + C_i \right) X_{l_i} \]

where

\[ X_{l_i} = \left( \frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}} \right)^{\frac{M}{2K}} \times \exp\left[ - \frac{G_i}{2} \right] \]

\[ \Delta t_{l_i} = \frac{1}{K} \ln\left( \frac{L_\infty - l_i}{L_\infty - l_{i+\Delta l}} \right) \]

and

\[ \Delta t_{l_i} = \exp\left( \frac{M \cdot \Delta t_{l_i} - G_i}{2} \right) \]

6. Finally, convert length to weight

\[ W_{l_i} = \alpha l_i^\beta \]

7. And calculate growth rate per length class

\[ G_i = \ln\left( \frac{W_{l_i + \Delta l}}{W_{l_i}} \right) \]
## LCA-The Steps

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Formula</th>
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<tbody>
<tr>
<td>1</td>
<td>Calculate $W$ from $l$</td>
<td>$W_{l_i} = \alpha \left( \frac{l_i + l_{i+\Delta l}}{2} \right)^\beta$</td>
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<tr>
<td>2</td>
<td>Calculate growth rate $G$</td>
<td>$G_{l_i} = \ln \left( \frac{W_{l_i+\Delta l}}{W_{l_i}} \right)$</td>
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<td>3</td>
<td>Calculate $\Delta t_{li}$</td>
<td>$\Delta t_{l_i} = \frac{1}{K} \ln \left( \frac{L_i - l_i}{L_i - l_{i+\Delta l}} \right)$</td>
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<tr>
<td>4</td>
<td>Calculate $X_l$</td>
<td>$X_{l_i} = \exp \left( \frac{M \cdot \Delta t_{l_i} - G_{l_i}}{2} \right)$</td>
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<tr>
<td>5</td>
<td>Estimate biomass in longest length interval</td>
<td>$B_{l_i} = C_{l_i} \cdot \frac{(M + F) \cdot \Delta t_{l_i} - G_{l_i}}{F \cdot \Delta t_{l_i}}$</td>
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<tr>
<td>6</td>
<td>Recursive equation</td>
<td>$B_{l_i} = (B_{l_i+\Delta l} \cdot X_{l_i} + C_{l_i}) \cdot X_{l_i}$</td>
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<td>7</td>
<td>Calculate $F$</td>
<td>$F_{l_i} \cdot \Delta t_{l_i} = \ln \left( \frac{B_{l_i}}{B_{l_i+\Delta l}} \right) - M \cdot \Delta t_{l_i} + G_{l_i}$</td>
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## Spreadsheet Calculation

### Constants

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### Intermediate Calculations

### Data

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**Total Biomass:** 1,782,677
Comparing Model Performance to Simulated Data
Model Results (based on B) vs. Simulated Data with no error
Jones Model (based on N) vs. Simulated Data with no error
Biomass-based estimation of $F_{x\%}$

Using Length Structure

The fishing mortality ($F_{x\%}$) that maintains the spawning biomass at an arbitrary percentage ($x\%$) of the virgin spawning biomass (i.e. $F=0$) can be determined by calculating the following ratio.

$$x\% = \frac{\text{Spawning Biomass with exploitation} (F_{x\%})}{\text{Virgin Spawning Biomass} (F = 0)}$$
Biomass-based estimation of $F_{x\%}$
Using Length Structure

Solving the following equation by using any nonlinear solution algorithm

\[
\frac{\sum_{i=1}^{l_\lambda} B'_i \cdot m_i \cdot e^{G_i - (M + F_{x\%}) \cdot S_i} \left( \frac{1}{K} \ln \left( \frac{(L_x - l_i)}{(L_x - l_{i+1})} \right) \right)}{\sum_{i=1}^{l_\lambda} B_i \cdot m_i \cdot e^{G_i - (M + F_{x\%}) \cdot S_i} \left( \frac{1}{K} \ln \left( \frac{(L_x - l_i)}{(L_x - l_{i+1})} \right) \right)} = x\% \]

where

\[
B_i = B_{i-1} \cdot e^{G_{i-1} - M \cdot \Delta_{i-1}} = B_{i-1} \cdot e^{G_{i-1} - M \cdot \frac{1}{K} \ln \left( \frac{(L_x - l_i)}{(L_x - l_{i+1})} \right)} \quad \text{for } F = 0
\]

\[
B'_i = B'_{i-1} \cdot e^{G_{i-1} - (M + F_{x\%}) \cdot S_{i-1} - M \cdot \frac{1}{K} \ln \left( \frac{(L_x - l_i)}{(L_x - l_{i+1})} \right)} \quad \text{for } F = x\%
\]

\[
G_i = \ln \left( \frac{W_{i+1}}{W_i} \right)
\]

$B_i$: Population biomass at length group $i$ when $F=0$.

$B'_i$: Population biomass at length group $i$ when $F=F_{x\%}$.

$m_i$: Maturity ratio of length group $i$.

$l_i$: Initial length of length group $i$.

$l_{i+1}$: Initial length of length group $i+1$ (or Maximum length of length group $i$).

$l_{\lambda}$: Last length group.

$F_{x\%}$: Fishing mortality that maintains the spawning biomass at $x\%$ of the virgin spawning biomass (or when $F=0$).

$S_i$: Fishing selectivity of length group $i$.

$G_i$: Growth rate of length group $i$.

Any Precautionary fishery metric

$F_{med}$, $F_{crit}$, $F_{lim}$, $F_{MSY}$, $F_{40\%}$, $F_{x\%}$
Estimation of Yield-per Recruit Using Length Structure

\[
\frac{Y}{R} = F \cdot W_\infty \cdot \exp[-M \cdot (t_c - t_r)] \cdot \sum_{n=0}^{3} \frac{U_n \cdot \exp[-n \cdot K \cdot (t_c - t_0)]}{F + M + n \cdot K}
\]

\(\alpha\) – length weight coefficient  
\(\beta\) – length weight power coefficient  
\(W_\infty\) – maximum weight  
\(L_\infty\) – maximum length  
\(F\) – fishing mortality  
\(M\) – natural mortality  
\(t_c\) – age of first capture  
\(t_r\) – age of recruitment  
\(t_0\) – von Bertalanffy parameter; \(t_0\) is the theoretical age at which the fish would have length zero if it had always grown as described by the von Bertalanffy equation  
\(K\) – von Bertalanffy growth rate  
\(l_c\) – length at first capture  
\(l_r\) – length at recruitment  
\(U_n\) – integration coefficient; \(U_0=1, U_1=-3, U_2=3, \text{ and } U_3=-1\)
Estimation of Yield-per Recruit Using Length Structure

Yield-per-Recruit Model Using Age Structure

\[
\frac{Y}{R} = F \cdot W_\infty \exp[-M \cdot (t_c - t_r)] \cdot \sum_{n=0}^{3} U_n \cdot \exp[-n \cdot K \cdot (t_c - t_0)] \cdot \frac{\sum_{n=0}^{3} U_n \cdot \exp[-n \cdot K \cdot (t_c - t_0)]}{F + M + n \cdot K}
\]

\[W_\infty = \alpha \cdot L_\infty^\beta\]

Yield-per-Recruit Model Using Length Structure

\[
\frac{Y}{R} = F \cdot (\alpha \cdot L_\infty^\beta) \cdot \left[ \frac{L_\infty - l_r}{L_\infty} \right]^{M/K} \cdot \left[ \frac{L_\infty - l_c}{L_\infty} \right]^{M/K} \cdot \sum_{n=0}^{3} U_n \cdot \left[ \frac{L_\infty - l_c}{L_\infty} \right]^n \cdot \frac{\sum_{n=0}^{3} U_n \cdot \left[ \frac{L_\infty - l_c}{L_\infty} \right]^n}{F + M + n \cdot K}
\]

\(\alpha\) – length weight coefficient
\(\beta\) – length weight power coefficient
\(W_\infty\) - maximum weight
\(L_\infty\) - maximum length
\(F\) – fishing mortality
\(M\) – natural mortality
\(t_c\) – age of first capture
\(t_r\) – age of recruitment
\(t_0\) – von Bertalanffy parameter; \(t_0\) is the theoretical age at which the fish would have length zero if it had always grown as described by the von Bertalanffy equation
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\(U_n\) – integration coefficient; \(U_0=1, U_1=-3, U_2=3, \) and \(U_3=-1\)
Estimation of Yield-per Recruit Using Length Structure

Yield-per-Recruit Model Using Age Structure

\[
\frac{Y}{R} = F \cdot W_\infty \cdot \exp[-M \cdot (t_c - t_r)] \cdot \sum_{n=0}^{3} \frac{U_n \cdot \exp[-n \cdot K \cdot (t_c - t_0)]}{F + M + n \cdot K}
\]

\[
\exp[-M \cdot (t_c - t_r)] = \exp[-M \cdot (t_0 - t_r)] \cdot \exp[-M \cdot (t_c - t_0)]
\]

\[
= \left[ \frac{L_\infty - l_r}{L_\infty} \right]^M \cdot \left[ \frac{L_\infty - l_c}{L_\infty} \right]^M
\]

\[
\frac{Y}{R} = F \cdot (\alpha \cdot L_\infty^\beta) \cdot \left[ \frac{L_\infty - l_r}{L_\infty} \right]^M \cdot \left[ \frac{L_\infty - l_c}{L_\infty} \right]^M \cdot \sum_{n=0}^{3} \frac{U_n \cdot \left[ \frac{L_\infty - l_c}{L_\infty} \right]^n}{F + M + n \cdot K}
\]

\[
\alpha \quad \text{– length weight coefficient}
\]
\[
\beta \quad \text{– length weight power coefficient}
\]
\[
W_\infty \quad \text{– maximum weight}
\]
\[
L_\infty \quad \text{– maximum length}
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\[
F \quad \text{– fishing mortality}
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M \quad \text{– natural mortality}
\]
\[
t_c \quad \text{– age of first capture}
\]
\[
t_r \quad \text{– age of recruitment}
\]
\[
t_0 \quad \text{– von Bertalanffy parameter; } t_0 \text{ is the theoretical age at which the fish would have length zero if it had always grown as described by the von Bertalanffy equation}
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\[
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l_c \quad \text{– length at first capture}
\]
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l_r \quad \text{– length at recruitment}
\]
\[
U_n \quad \text{– integration coefficient; } U_0=1, U_1=-3, U_2=3, \text{ and } U_3=-1
\]

Yield-per-Recruit Model Using Length Structure
Estimation of Yield-per Recruit Using Length Structure

Yield-per-Recruit Model Using Age Structure

$$\frac{Y}{R} = F \cdot W_\infty \cdot \exp[- M \cdot (t_c - t_r)] \cdot \sum_{n=0}^{3} \frac{U_n}{F + M + n \cdot K} \cdot \exp[- n \cdot K \cdot (t_c - t_0)]$$

$$\exp[- n \cdot K \cdot (t_c - t_0)] = \left[ \frac{L_\infty - l_c}{L_\infty} \right]^n$$

Yield-per-Recruit Model Using Length Structure

$$\frac{Y}{R} = F \cdot (\alpha \cdot L_\infty^{\beta}) \cdot \left[ \frac{L_\infty - l_c}{L_\infty} \right]^M \cdot \left[ \frac{L_\infty - l_r}{L_\infty} \right]^M \cdot \sum_{n=0}^{3} \frac{U_n}{F + M + n \cdot K}$$

α = length weight coefficient
β = length weight power coefficient
$W_\infty$ = maximum weight
$L_\infty$ = maximum length
F = fishing mortality
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t_c = age of first capture
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t_0 = von Bertalanffy parameter; $t_0$ is the theoretical age at which the fish would have length zero if it had always grown as described by the von Bertalanffy equation
K = von Bertalanffy growth rate
l_c = length at first capture
l_r = length at recruitment
U_n = integration coefficient; $U_0 = 1$, $U_1 = -3$, $U_2 = 3$, and $U_3 = -1$
Yield Contours with $F_{40\%}$ and $F_{\text{max}}$ Isopleths

For any length at first capture ($L_c$), $F_{40\%}$ is always less than $F_{\text{max}}$.

$F_{40\%}$ maximizes yield.
Apply model to eastern Bering Sea northern rock sole data
Eastern Bering Sea
Northern Rock Sole

Biomass (mmt) vs. Year

- Biomass in 2003 is significantly higher than in other years.
Eastern Bering Sea
Northern Rock Sole

Year
2003 2004 2005 2006 2007

Biomass (mmt)

Model
Survey
LCA Biomass
Eastern Bering Sea
Northern Rock Sole

Biomass (mmt)

Year

2003 2004 2005 2006 2007

Model
Survey
LCA Biomass
Eastern Bering Sea
Northern Rock Sole

![Graph showing biomass in the Eastern Bering Sea from 2003 to 2007. The x-axis represents the year, and the y-axis represents biomass (mmt). The graph includes data from Model, Survey, and LCA Biomass models.]
Eastern Bering Sea
Northern Rock Sole

Biomass (mmt)

Year

2003 2004 2005 2006 2007

Model
Survey
LCA Biomass
Eastern Bering Sea
Northern Rock Sole

![Graph showing population dynamics](image)
Biomass and Fishing Mortality
Northern Rock Sole

Body Size (FL, cm)

Biomass ($10^3$ mt)

Fishing Mortality
Conclusions

- Biomass LCA is simple to apply
  - Assumptions are minimal
  - Calculations are not complicated and easily implemented with spreadsheet software
  - Data needs are modest - only 1 year of catch information

- Method works well compared to simulated data with known properties

- Can be easily extended to include calculation of useful and relevant management metrics
  - Biomass and fishing mortality by length
  - Population biomass
  - $F_{x\%}$ calculations and biomass estimates allow calculation of approximate ABC or TAC
  - Yield per Recruit using length structure

- Biomass LCA should be considered for small scale fisheries resource assessment